ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E/B.Tech END SEMESTER EXAMINATIONS - NOV/DEC 2024

Semester III

MA23C06 - PARTIAL DIFFERENTIAL EQUATIONS AND COMPLEX FUNCTIONS

(Regulation 2023)

Time: 3 Hours | Answer ALL Questions | Max. Marks:100

CO1	Understand the concepts of partial differential equations in practical situations.
CO2	Obtain the solutions of the partial differential equations using Fourier series.
CO3	Understand the Concepts of complex functions in practical situations.
CO4	Understand the conformal mapping and its applications.
CO5	Apply the complex integrations in engineering problems.

BL - Bloom's Taxonomy Levels

L1 - Remembering; L2 - Understanding; L3 - Applying; L4 - Analysing; L5 - Evaluating; L6 - Creating

Q.No.	Question	Marks	CO	BL
	PART - A ($10 \times 2 = 20$ Marks)			
1.	Solve $xdx + ydy = z$.	2	1	L2
2.	Find the complete integral of $p + q = pq$	2	1	L2
3.	Classify the PDE: $5u_{xx} + 4u_{yy} + 3u_y - 2u_x = 0$	2	2	L2
4.	What is the general solution of one dimensional heat flow equation in steady state conditions.	2	2	L2
5.	State C-R equation for the complex function.	2	3	L1
6.	True or False: If u and v are harmonic, then $u + iv$ is analytic. Justify your answer.	2	3	L2
7.	Find z if $e^z = 1 + i$.	2	4	L2
8.	Under what condition of a and b, the map $f(z) = \dfrac{z+a}{z+b}$ is bilinear.	2	4	L1
9.	Define the essential singularity of the complex function $f(z)$.	2	5	L1
10.	Can you conclude that $\int_C Re(z)dz = 0$ over $C : \lvert z \rvert = 1$ by using Cauchy theorem. Justify your answer	2	5	L2
	PART - B ($5 \times 13 = 65$ Marks)			
11.(a)	**(i)** Find the general solution of PDE $\left(\dfrac{y-z}{yz}\right)p + \left(\dfrac{z-x}{zx}\right)q = \dfrac{x-y}{xy}$	8	1	L3
	(ii) Form the partial differential equation by eliminating the arbitrary costants a and b from $z = xy + y\sqrt{x^2 - a^2} + b$	5	1	L3
	(OR)			

Q.No.	Question	Marks	CO	BL
11.(b)	(i) Solve $(D^2+2DD'+D'^2)z=\sin(x-y)+e^{y-x}$	8	1	L3
	(ii) Solve $p(1-q^2)=q(1-z)$	5	1	L3
12.(a)	(i) A tightly stretched string with fixed end points $x=0$ and $x=\ell$ is initially in a position given by $y(x,0)=y_0\sin\frac{\pi x}{\ell}$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time t.	13	2	L4
	(OR)			
12.(b)	(i) A square plate is bounded by the lines $x=0, y=0, x=20$ and $y=20$. Its faces are insulated. The temperature along the upper horizondal edge is given by $u(x,20)=x(20-x),\ 0<x<20$ while the other two edges are kept at $0°C$. Find the steady state temperature distribution in the plate	13	2	L4
13.(a)	If $f(z)$ is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2=4\|f'(z)\|^2$.	8	3	L3
	(ii) Test the analyticity of the function $f(z)=x^2-y^2-i\left(\frac{y}{x^2+y^2}\right)$.	5	3	L3
	(OR)			
13.(b)	(i) Check whether the function $u(x,y)=e^x[x\cos y-y\sin y]$ is harmonic. If so, find its harmonic conjugate $v(x,y)$.	8	3	L3
	(ii) Prove that (i) $\|e^z\|=e^x$ (ii) $\sin^2 z+\cos^2 z=1$	5	5	L4
14.(a)	(i)Find the bilinear mapping $w=f(z)$ which maps $0,2,i$ in the z-plane into $-1,3,-i$ of the w-plane. Also find the image of the region $\|z-1\|<1$ under $w=f(z)$and compute the fixed points of $f(z)$.	13	4	L4
	(OR)			
14.(b)	(i) Find the image of the strip $1\le x\le 2$ under the map $w=\frac{1}{z}$	7	4	L4
	(ii)Find the image of the region $1\le x\le 2\ \ 0\le y\le\pi$ under the map $w=e^z$.	6	4	L4
15.(a)	(i)Evaluate $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ by contour integration.	8	5	L4
	(ii)Find the residue at $z=0$ for the function $f(z)=\frac{1+ze^{-\frac{1}{z}}}{z}$.	5	5	L4
	(OR)			
15.(b)	(i) Evaluate $\int_c\frac{z^2-3z+2}{(z-1)^2(z+2)}dz$ where c is the circle (i) $\|z\|=\frac{1}{2}$ (ii) $\|z\|=\frac{3}{2}$ (iii) $\|z-1\|=\pi$.	8	5	L4
	(ii) Classify the singularities of the function $f(z)=\frac{z^3-3z^2+2z}{(z-1)^2(z+2)}$	5	5	L4
	PART - C ($1\times 15=15$ Marks)			
16	(i) A rod 60 cm long has its ends A and B kept at 20°C and 80°C respectively until steady-state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find $u(x,t)$.	15	2	L5